How big should the field of view be?

■ Mapping speed

Assume we're talking about point sources and we're talking about background-limited observations. Denote the seeing FWHM by σ. If we forget about AO PSFs having different Strehls than natural seeing PSFs, then the signal-to-noise ratio ought to scale as $1/\sigma$ and integration time goes as $1/\sigma^2$.

Say we're trying to map out a bunch of sources over a solid angle ω to a given SNR. Let's compare the speed for completing the observations as a function of field of view.

Case 1: Natural seeing (σ=0.4"), 30' diameter FOV. Say that to cover ω takes time *T*.

Case 2: Ground-layer AO (GLAO). Assume σ=0.2" (from David's sims), 11' diameter FOV (maximum possible at Cass with Gemini). The time to cover ω would go as

In[1]:= $$\mathbf{T} \times \left(\frac{\mathbf{0.2\ arcsec}}{\mathbf{0.4\ arcsec}}\right)^2 \times \left(\frac{\mathbf{30\ arcmin}}{\mathbf{11\ arcmin}}\right)^2$$

Out[1]= 1.8595 T

So what looks like a factor of 9 gain in area translates to a factor of < 2 gain in mapping speed. Putting an instrument at prime is hugely expensive (new top end) and putting in an f/6 secondary ain't cheap either, so there needs to be a really really good reason to do either thing. Seems to me that mapping speed isn't it.

Case 3: Multi-conjugate AO (MCAO). Assume σ=0.05" (diffraction limit in H for 8.1m aperture telescope), and 2' diameter FOV (the current Gemini MCAO plan I think). The time to cover ω goes as

In[2]:= $$\mathbf{T} \times \left(\frac{\mathbf{0.05\ arcsec}}{\mathbf{0.4\ arcsec}}\right)^2 \times \left(\frac{\mathbf{30\ arcmin}}{\mathbf{2\ arcmin}}\right)^2$$

Out[2]= 3.51563 T

So MCAO is less efficient than GLAO though they're close, and it looks like MCAO only a factor of 4 less efficient than the natural seeing instrument even though it's got a factor of 225 smaller area.

■ Detectors needed

Case 1: Natural seeing. To critically sample the σ=0.4" PSF at the Nyquist frequency one needs this many pixels to span the FOV:

In[3]:= $30\ \text{arcmin} \times 60\ \frac{\text{arcsec}}{\text{arcmin}} \Big/ (0.4\ \text{arcsec} / 2.2)$

Out[3]= 9900.

So we're talking an 5 2K devices to span the FOV, or of order 20 2K devices to cover the circular area.

Case 2: GLAO.

In[4]:= $11\ \text{arcmin} \times 60\ \frac{\text{arcsec}}{\text{arcmin}} \Big/ (0.2\ \text{arcsec} / 2.2)$

Out[4]= 7260.

Since

In[5]:= $(7260. / 9900)^2$

Out[5]= 0.537778

We'd need about half the number of detectors relative to the natural seeing instrument.

Case 3: MCAO.

In[6]:= $2\ \text{arcmin} \times 60\ \frac{\text{arcsec}}{\text{arcmin}} \Big/ (0.05\ \text{arcsec} / 2.2)$

Out[6]= 5280.

Since

In[7]:= $(5280. / 9900)^2$

Out[7]= 0.284444

The number of detectors for the MCAO instrument is relatively small – only 25% of what's needed for the 30' FOV instrument, and maybe 4 2K's would do.